Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

9 May 2023

Zone A afternoon │ **Zone B** morning │ **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The cities Lucknow (L), Jaipur (J) and Delhi (D) are represented in the following diagram. Lucknow lies $500 \, \text{km}$ directly east of Jaipur, and $\hat{JLD} = 25°$.

diagram not to scale
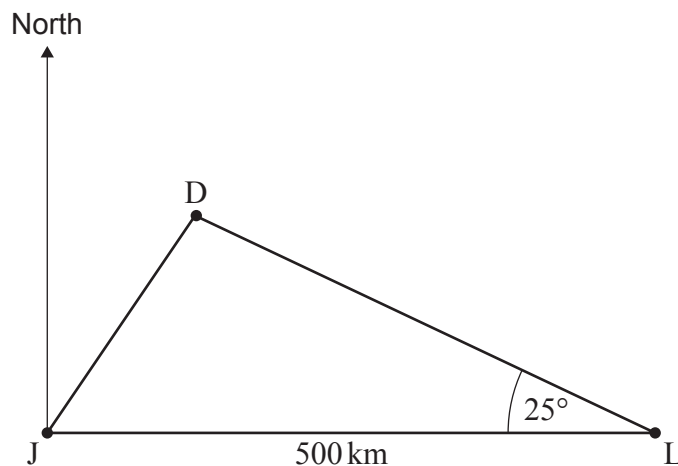
The bearing of D from J is $034°$.

(a)   Find $\hat{JDL}$. [2]

(b)   Find the distance between Lucknow and Delhi. [3]

2. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in $\text{USD}(\$)$.

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested $\$15\,000$ in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra $\$M$.

(c) Find the value of M. [3]

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3. [Maximum mark: 5]

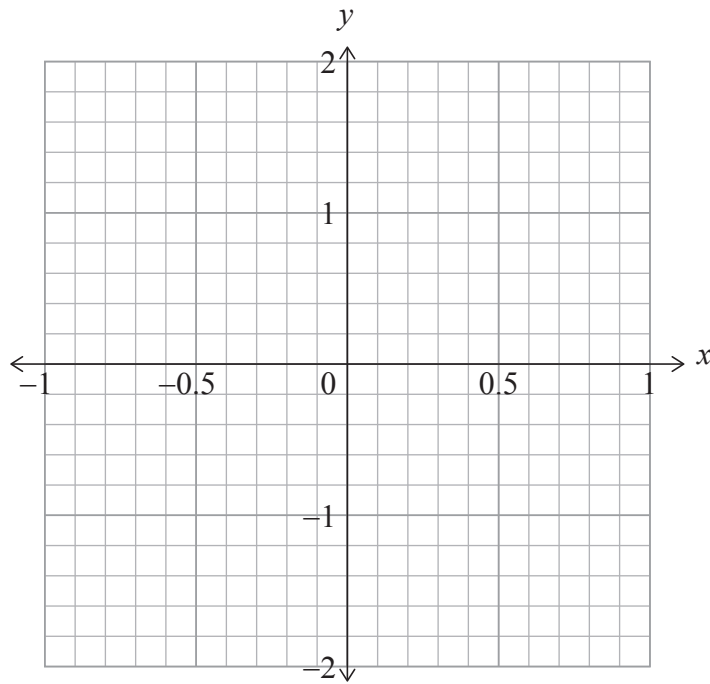The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \leq x \leq 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]

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4. [Maximum mark: 7]

The total number of children, y, visiting a park depends on the highest temperature, T, in degrees Celsius (°C). A park official predicts the total number of children visiting his park on any given day using the model $y = -0.6T^2 + 23T + 110$, where $10 \le T \le 35$.

(a) Use this model to estimate the number of children in the park on a day when the highest temperature is $25\,°\text{C}$. [2]

An ice cream vendor investigates the relationship between the total number of children visiting the park and the number of ice creams sold, x. The following table shows the data collected on five different days.

Total number of children (y)	81	175	202	346	360
Ice creams sold (x)	15	27	23	35	46

(b) Find an appropriate regression equation that will allow the vendor to predict the number of ice creams sold on a day when there are y children in the park. [3]

(c) Hence, use your regression equation to predict the number of ice creams that the vendor sells on a day when the highest temperature is $25\,°\text{C}$. [2]

5. [Maximum mark: 5]

A company manufactures metal tubes for bicycle frames. The diameters of the tubes, $D\,\text{mm}$, are normally distributed with mean 32 and standard deviation σ. The interquartile range of the diameters is 0.28.

Find the value of σ.

6. [Maximum mark: 7]

The coefficient of x^6 in the expansion of $(ax^3 + b)^8$ is 448.

The coefficient of x^6 in the expansion of $(ax^3 + b)^{10}$ is 2880.

Find the value of a and the value of b, where a, $b > 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 13]

The temperature of a cup of tea, t minutes after it is poured, can be modelled by $H(t) = 21 + 75e^{-0.08t}$, $t \geq 0$. The temperature is measured in degrees Celsius (°C).

(a)   (i)   Find the initial temperature of the tea.

       (ii)   Find the temperature of the tea three minutes after it is poured.   [2]

(b)   Write down the value of $H'(3)$.   [2]

(c)   Interpret the meaning of your answer to part (b) in the given context.   [2]

(d)   After k minutes, the tea will be below $67\,°C$ and cool enough to drink.

       Find the least possible value of k, where $k \in \mathbb{Z}^+$.   [3]

As the tea cools, $H(t)$ approaches the temperature of the room, which is constant.

(e)   Find the temperature of the room.   [2]

(f)   Find the limit of $H'(t)$ as t approaches infinity.   [2]

Do **not** write solutions on this page.

8. [Maximum mark: 17]

In a large city, 160 people were surveyed. Of those, 60 were children (C) and the rest adults (A).

Each person in the survey was asked whether they preferred milk chocolate (M) or dark chocolate (D). It was found that 48 of the children preferred milk chocolate. This information is shown in the following table.

	M (milk chocolate)	**D (dark chocolate)**	**Total**
C (children)	48	p	60
A (adults)	x	y	q

(a) Find the value of

   (i) p;

   (ii) q. [2]

(b) Three people are chosen at random from those surveyed. Find the probability that all three are adults. [4]

(c) (i) Given that $\mathrm{P}(A \mid M) = \dfrac{1}{3}$, find the value of x.

   (ii) A person is chosen at random from those surveyed. Write down the probability that they are an adult who prefers milk chocolate. [4]

(d) Determine if the events A and M are independent. Justify your answer. [3]

It can be assumed that the survey results are representative of the population of the city.

(e) Ten people in the city are chosen at random. Find the probability that at least five of them prefer dark chocolate. [4]
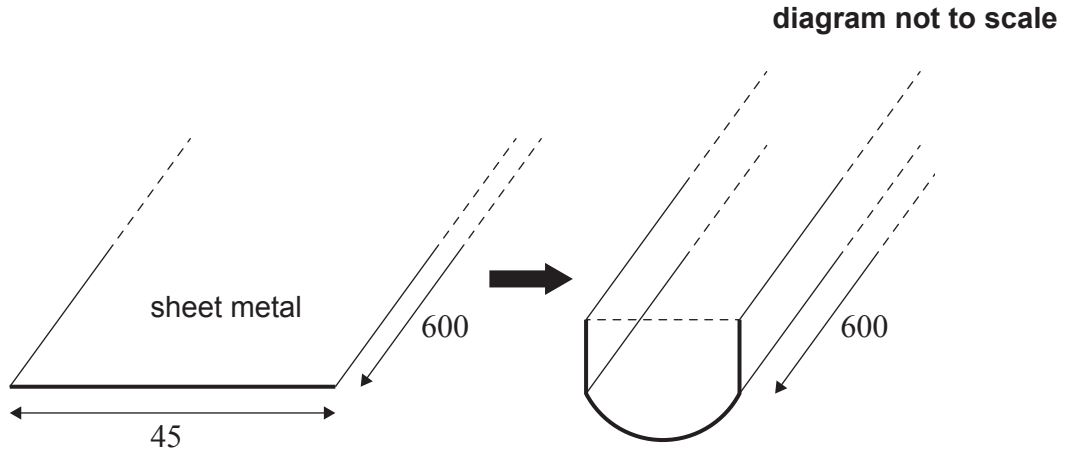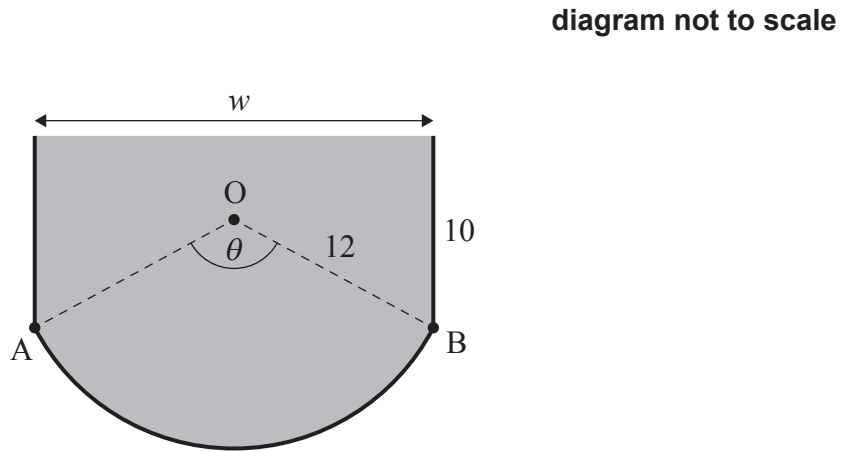
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9.   [Maximum mark: 15]

An engineer is designing a gutter to catch rainwater from the roof of a house.

The gutter will be open at the top and is made by folding a piece of sheet metal $45\,\text{cm}$ wide and $600\,\text{cm}$ long.

diagram not to scale

The cross-section of the gutter is shaded in the following diagram.

diagram not to scale

The height of both vertical sides is $10\,\text{cm}$. The width of the gutter is $w\,\text{cm}$.

Arc AB lies on the circumference of a circle with centre O and radius $12\,\text{cm}$.

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(Question 9 continued)

Let $A\hat{O}B = \theta$ radians, where $0 < \theta < \pi$.

(a) Show that $\theta = 2.08$, correct to three significant figures. [3]

(b) Find the area of the cross-section of the gutter. [7]

In a storm, the total volume, in cm^3, of rainwater that enters the gutter can be modelled by a function $R(t)$, where t is the time, in seconds, since the start of the storm.

It was determined that the **rate** at which rainwater entered the gutter could be modelled by

$$R'(t) = 50\cos\left(\frac{2\pi t}{5}\right) + 3000, \; t \geq 0.$$

During any 60-second period, if the volume of rainwater entering the gutter is greater than the volume of the gutter, it will overflow.

(c) Determine whether the gutter overflowed in this storm. Justify your answer. [5]

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Answers written on this page
will not be marked.